SLOPE FIELDS

slope field is a graphical display which shows the flow of tangent lines to the family of solution curves of a differential equation. Often, this flow diagram provides valuable information about the nature of the solution curve, i.e., whether it is polynomial, exponential, circular, trigonometric, etc. Slope fields can be constructed over a region of the plane by direct substitution into the differential equation, or they can be generated using the graphing calculator and an appropriate program.

There are generally two types of problems that involve slope fields. In the first, we are given a differential equation and asked to produce its slope field diagram. In the second, we are given a slope field and asked to match it to a particular differential equation which has those characteristics.

CONSTRUCTING A SLOPE FIELD.

When constructing slope fields, it is helpful to first create a table of values for a specific interval of x and y. For example, if we know that $dy/dx = xy/4$ for x in [-2,2] and y in [-2,2], we can construct a grid to evaluate the slopes at each integer pair of values. Complete the grid of slope values for this differential equation:

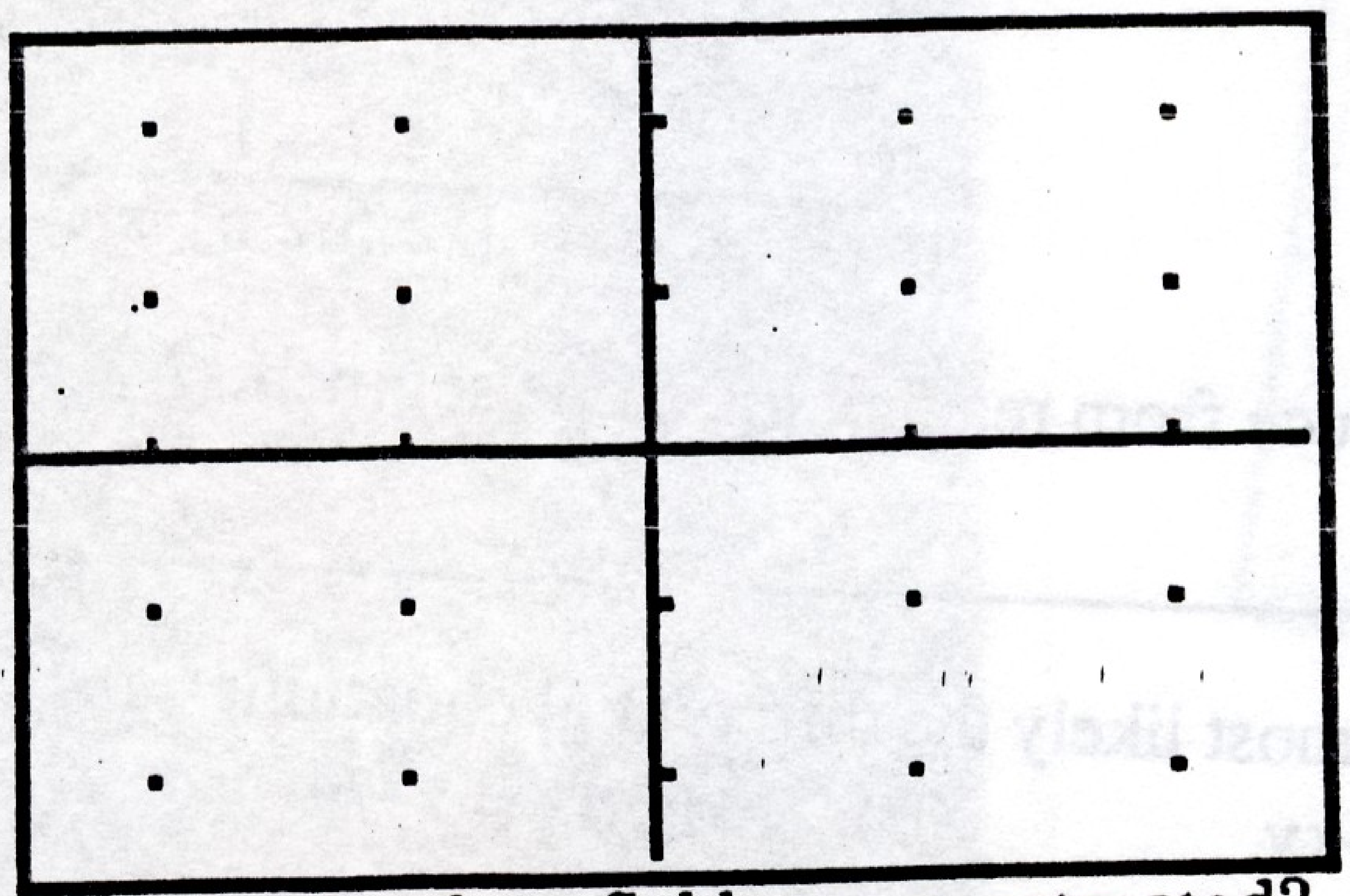
1.

x values

y values	-2	-1	0	1	2
2					
1					
0					
- 1					
- 2					

Once the grid is complete, the slope segments can be drawn through the lattice points of a graph. Draw the slope field for the values obtained above:

2.



3. What kind of function seems to be pictured by the slope field you constructed?

READING A SLOPE FIELD.

When reading a slope field, it is important to look for clues the slope segments give us about the behavior of the differential equation and, by extension, its family of solutions. It is possible to read a slope field one segment at a time; however, this can be exhausting for very large fields. It is probably easier in many cases to spot trends in the slope field that tell us something about how x and y are related in the differential equation. Here are some approaches you can use:

- Examine slope field segments along vertical lines. If the segments along each vertical line have the same slope, then the differential equation does not depend on y, because, as y varies, the slope does not.

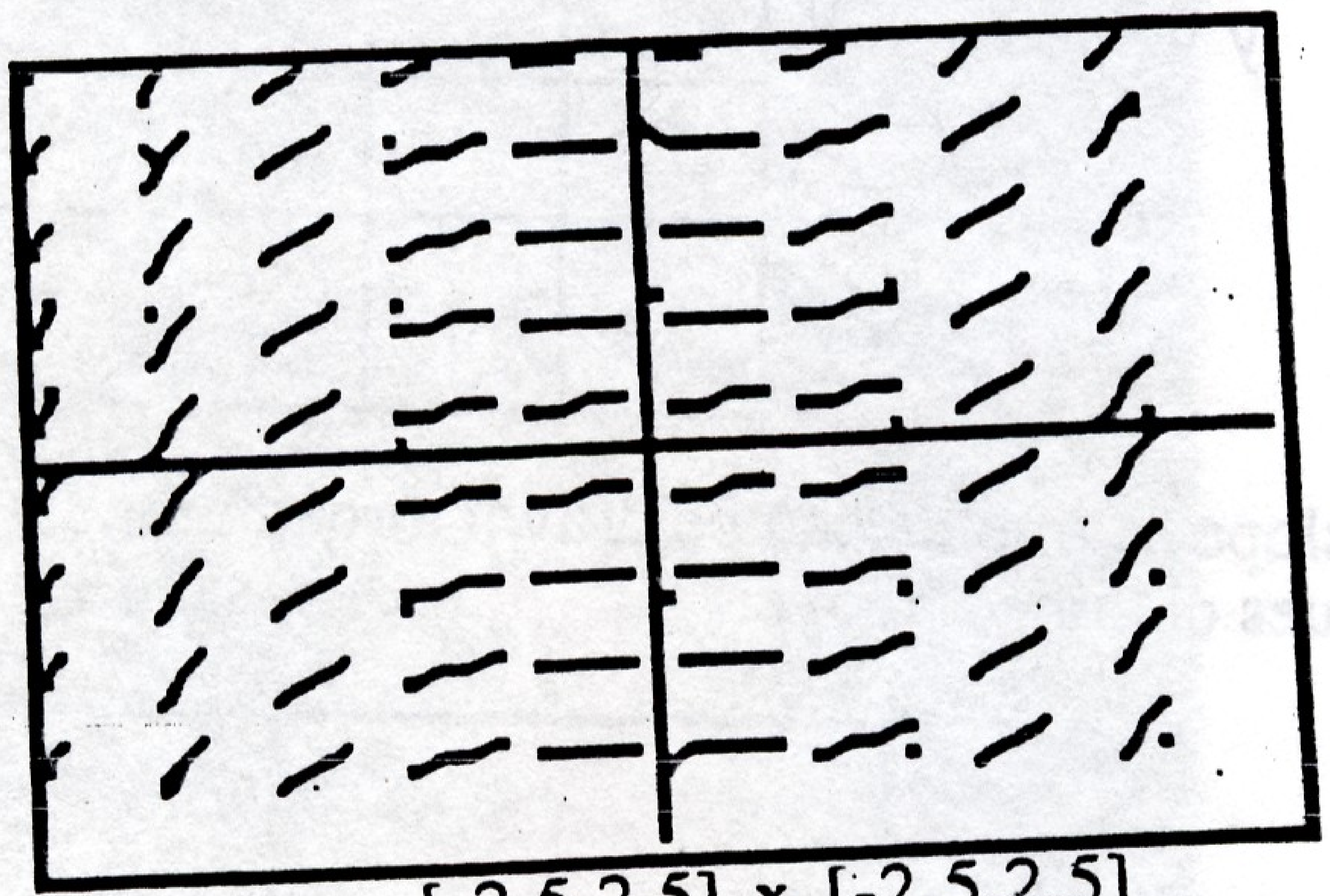
- Examine slope field segments along horizontal lines. If the segments along each horizontal line have the same slope, then the differential equation does not depend on x.

- Examine slope field segments in the first quadrant. If the segments have positive slope, then there are likely no negatives in the expression of the differential equation. If the slopes become larger as x gets larger, then dy/dx varies directly with x; likewise for y. Otherwise, we can determine that the slope is inversely related to one or both variables.

- If the slope field evinces a curve which looks familiar, check by differentiating that curve to see if its slope field fits the graphical data.

NOTE: There are occasional anomalies in the appearance of slope fields, due to the way they are generated on a calculator. Small discrepancies in slope can usually be dismissed.

Consider the following slope field:



[-2.5,2.5] x [-2.5,2.5]

4. What can you deduce from reading the slope segments? ______________________

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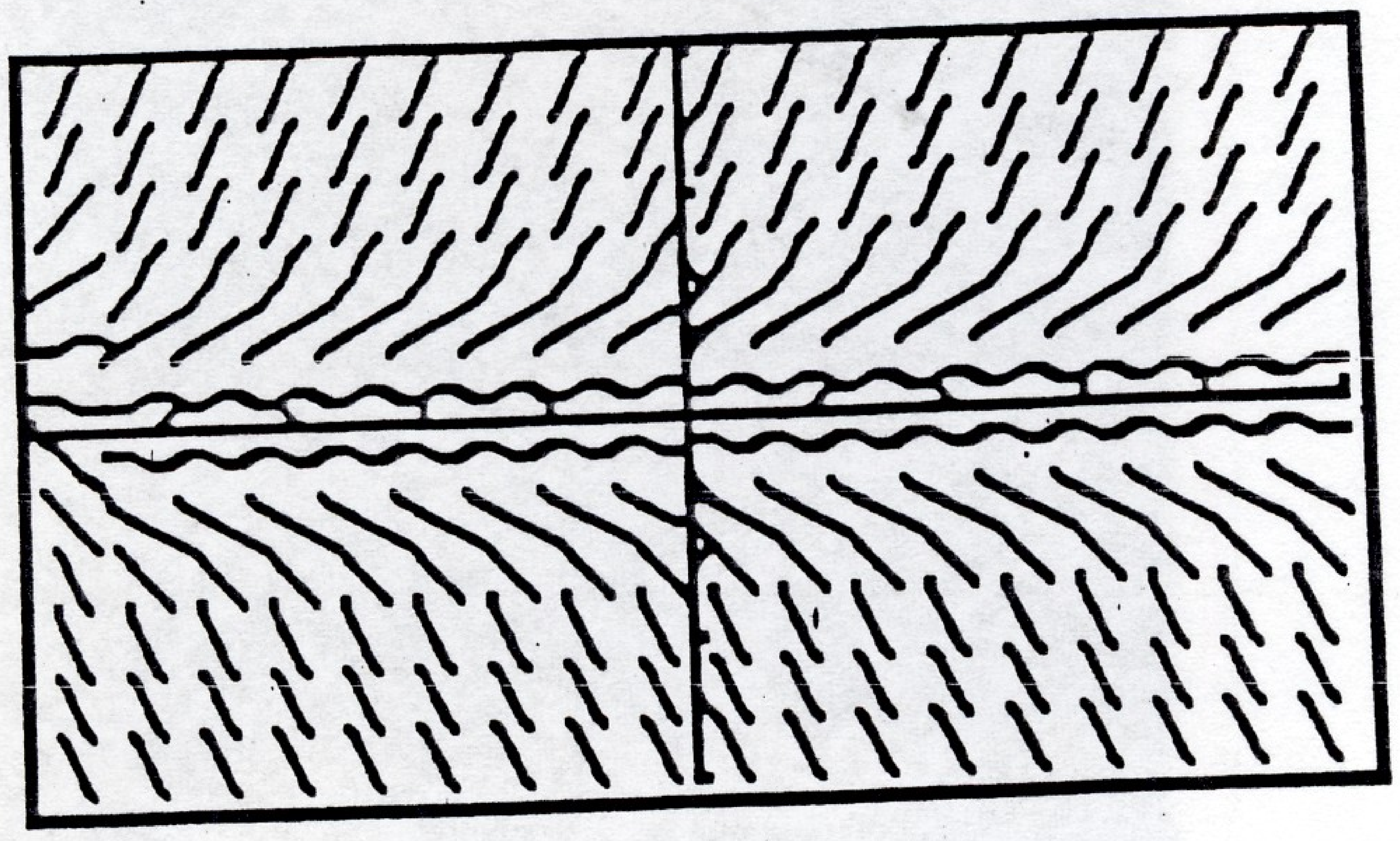
5. Which of these is most likely the differential equation:

(A) $dy/dx = .5xy$ (B) $dy/dx = x^2/y$ (C) $dy/dx = .5x^2$

e are some more slope fields to practice on. In each, match the slope field with its differential ation.

5. Which of the following differential equations has the solution slope field pictured at right?



(A) $dy/dx = .5y$

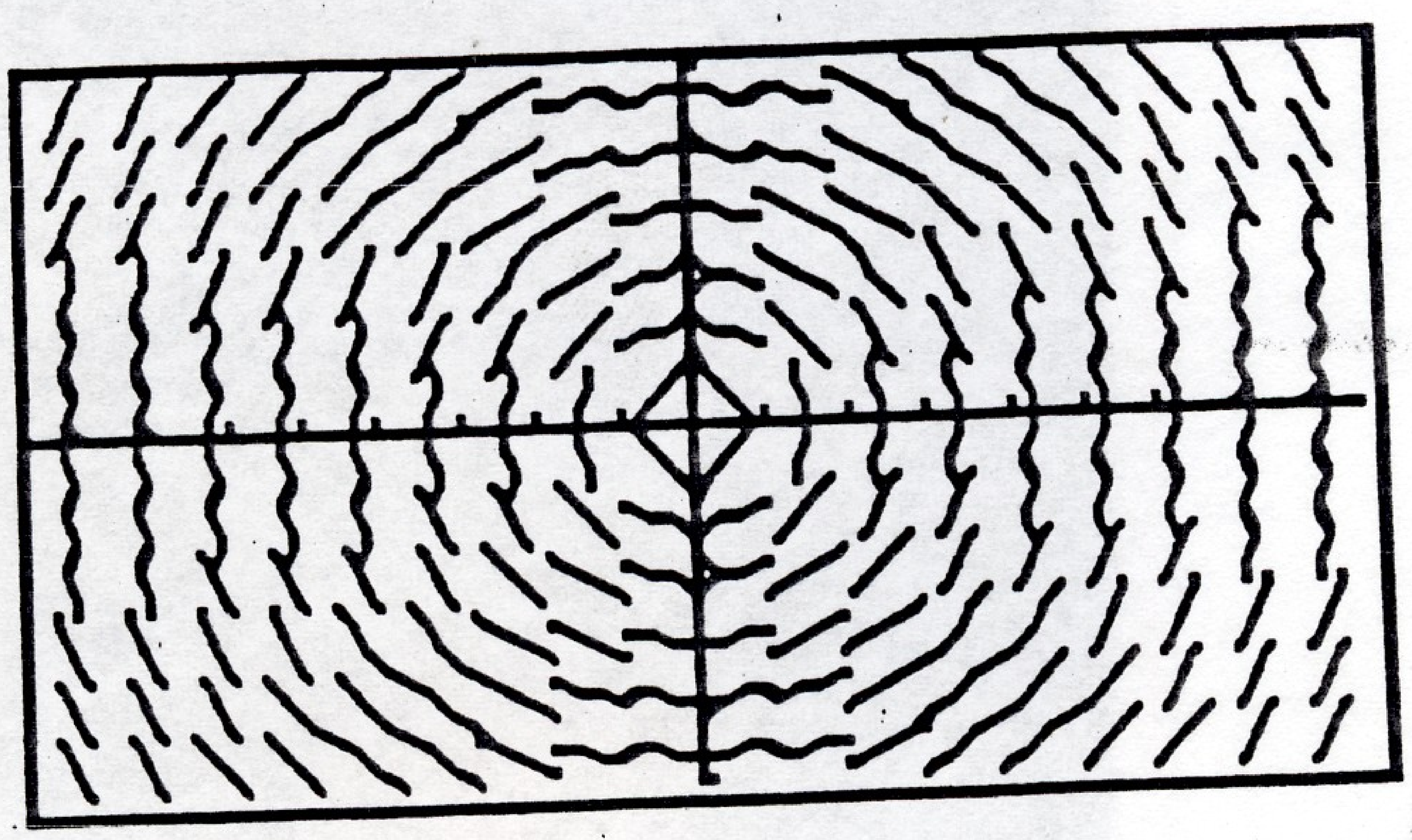
(B) $dy/dx = .2x/y$

(C) $dy/dx = xy$

(D) $dy/dx = x + y$

(E) $dy/dx = 1/x$

7. Which of the following differential equations has the solution slope field pictured at right?



(A) $dy/dx = x^2$

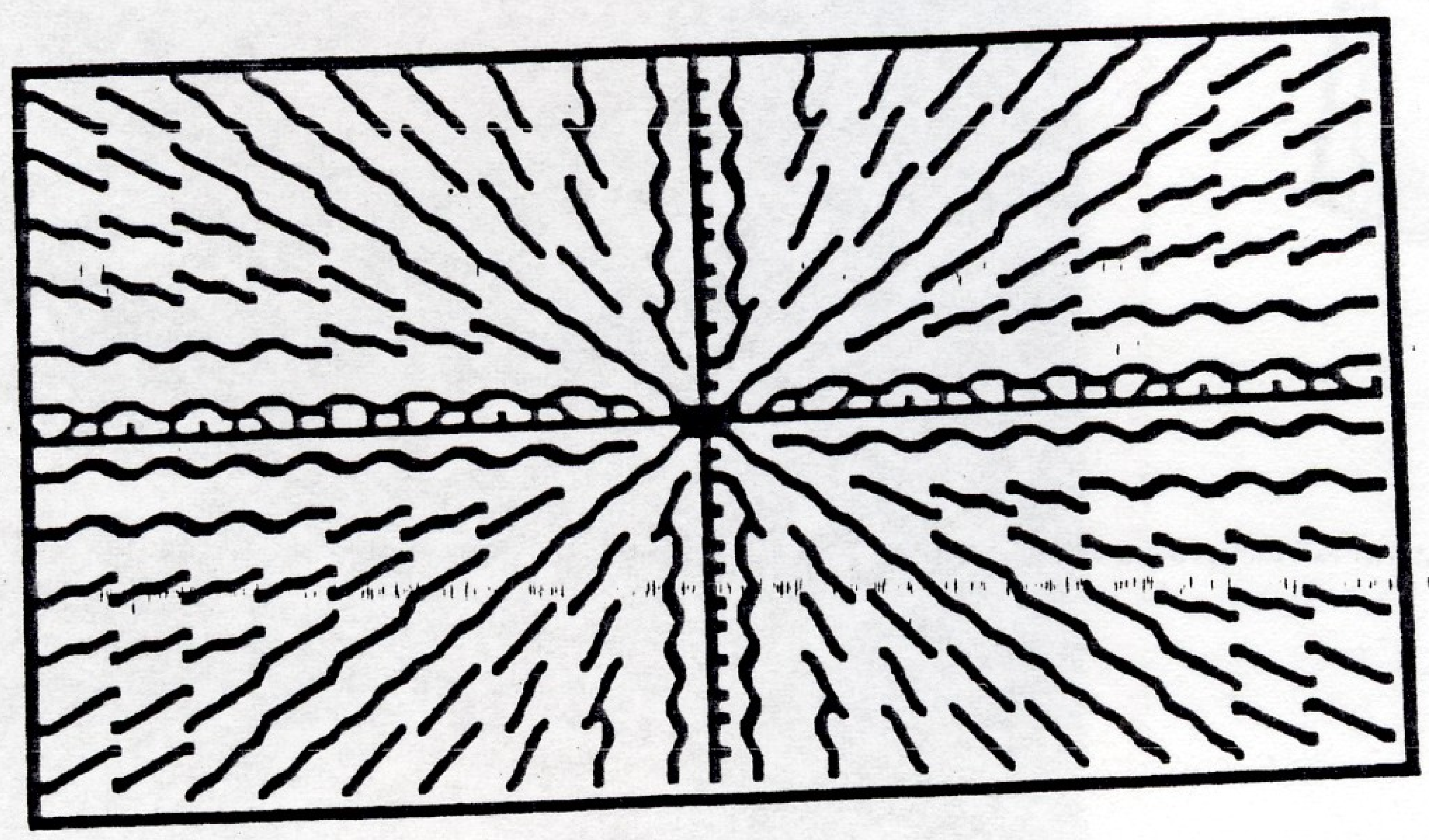
(B) $dy/dx = y/x$

(C) $dy/dx = -y$

(D) $dy/dx = -x/y$

(E) $dy/dx = x^2 + y^2$

8. Which of the following differential equations has the solution slope field pictured at right?



(A) $dy/dx = x + y$

(B) $dy/dx = x - y$

(C) $dy/dx = x^2$

(D) $dy/dx = 2y$

(E) $dy/dx = y/x$